PHASE TRANSITION IN SCALAR THEORY
QUANTIZED ON THE LIGHT-FRONT[a]

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Abstract: *The renormalization of the two dimensional light-front quantized ϕ^4 theory is discussed. The mass renormalization condition and the renormalized constraint equation are shown to contain all the information to describe the phase transition in the theory, which is found to be of the second order. We argue that the same result would also be obtained in the conventional equal-time formulation.*

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1. We use the light-front framework [1] to study the stability of the vacuum in the ϕ^4 theory when the coupling constant is increased from vanishingly small values to larger values. The hamiltonian formulation now has an additional ingredient in the form of a constraint eq. It is not convenient to eliminate the constraint; it would require to handle a nonlocal hamiltonian to construct renormalized theory. We could alternatively perform the ususal renormalization and obtain instead a *renormalized constraint eq.*. This along with, say, the mass renormalization condition may furnish important information on the nature of the phase transition in the theory. We recall that there are rigorous proofs on the triviality of ϕ^4 theory in the continuum for more than four space-time dimensions and on its interactive nature for dimensions less than four. In the important case of four dimensions the situation is still unclear and light-front dynamics may throw some light on it. In view of the complexity of the renormalization problem in this case we will illustrate our points by considering only the two dimensional theory, which is of importance in the condensed matter physics. For example, from the well established results on the generalized Ising models, Simon and Griffiths [2] conjectured some time ago that the two dimensional ϕ^4 theory should show the *second order* phase transition. We do confirm it here perhaps for the first time. The earlier works [3] based on variational methods usually in the conventional equal-time formulation, viz., the Hartree approximation, Gaussian effective potential, using a scheme of canonical transformations etc., give rise to the first order phase transition contradicting the conjecture.

2. We renormalize the theory based on (3-1) and (4-1) of ref. [1] without solving the constraint eq. (4-1) first to eliminate ω. We set $M_0^2(\omega) = (m_0^2 + 3\lambda\omega^2)$ and choose $\mathcal{H}_0 = {M_0}^2\varphi^2/2$ so that $\mathcal{H}_{int} = \lambda\omega\varphi^3 + \lambda\varphi^4/4$. In view of the superrenormalizability of the two dimensional theory we need to do only the mass renormalization. We assume that the bare mass m_o is nonvanishing and set $\int dx\varphi(x,\tau) = \sqrt{2\pi}\,\tilde{\varphi}(k=0,\tau) = 0$, e.g., $k \equiv k^+ > 0$. We follow [3] the straightforward Dyson-Wick expansion based on the Wick theorem in place of the old fashioned perturbation theory.

The self-energy correction to the *one loop order* is $-i\Sigma(p) = -i\Sigma_1 - i\Sigma_2(p) = (-i6\lambda)\frac{1}{2}D_1({M_0}^2) + (-i6\lambda\omega)^2\frac{1}{2}(-i)D_2(p^2,{M_0}^2)$, where the divergent contribution D_1

refers to the one-loop tadpole while D_2 to the one-loop *finite contribution* coming from the φ^3 vertex. The latter carries the sign *opposite* to that of the first and it will be argued below to be of the same order in λ as the first one. Using the dimensional regularization we obtain

$$M_0{}^2(\omega) = M^2(\omega) + \frac{3\lambda}{4\pi}\Big[\gamma + ln(\frac{M^2(\omega)}{4\pi\mu^2})\Big] + 18\lambda^2\omega^2 D_2(p, M^2)|_{p^2=-M^2} + \frac{3\lambda}{2\pi}\frac{1}{(n-2)}. \quad (1)$$

Here $M(\omega)$ is the physical mass and we take into account that in view of the tree level result $\omega(\lambda\omega^2 + m_0{}^2) = \omega[M_0{}^2(\omega) - 2\lambda\omega^2] = 0$ the correction term $\lambda^2\omega^2$, for $\omega \neq 0$, is really of the first order in λ. We ignore terms of order λ^2 and higher and remind that M_0 depends on ω which in its turn is involved in the constraint eq. From (1) we obtain the *mass renormalization condition*

$$M^2 - m^2 = 3\lambda\omega^2 + \frac{3\lambda}{4\pi} ln(\frac{m^2}{M^2}) - \lambda^2\omega^2\frac{\sqrt{3}}{M^2} \quad (2)$$

where $M(\omega) \equiv M$ and $M(\omega = 0) \equiv m$ indicate the phsyical masses in the *asymmetric* and *symmetric* phases respectively.

We next take the vacuum expectation value of the constraint eq. (4-1) in order to obtain another independent equation. To the lowest order we find *renormalized constraint eq.*

$$\beta(\omega) \equiv \omega\Big[M^2 - 2\lambda\omega^2 + \lambda^2\omega^2\frac{\sqrt{3}}{M^2} - \frac{6\lambda^2}{(4\pi)^2}\frac{b}{M^2}\Big] = 0. \quad (3)$$

where b arises from D_3, a finite integral like D_2 with three denominators, and $b \simeq 7/3$.

The expression for *difference* of the renormalized *vacuum energy density*, in the equal-time formulation, in the broken and the symmetric phases, is found to be finite and given by

$$\begin{aligned} F(\omega) &= \mathcal{E}(\omega) - \mathcal{E}(\omega = 0) \\ &= \frac{(M^2 - m^2)}{8\pi} + \frac{1}{8\pi}(m^2 + 3\lambda\omega^2)\, ln(\frac{m^2}{M^2}) + \frac{3\lambda}{4}\Big[\frac{1}{4\pi} ln(\frac{m^2}{M^2})\Big]^2 \\ &\quad + \frac{1}{2}m^2\omega^2 + \frac{\lambda}{4}\omega^4 + \frac{1}{2!}.(-i6\lambda\omega)^2.\frac{1}{6}.D_3(M), \end{aligned} \quad (4)$$

which is also independent of the arbitrary mass μ when we use the mass renormalization condition. We verify that $(dF/d\omega) = \beta$ and $d^2F/d\omega^2 = \beta'$ and except for the finite last term it coincides with the result in the earlier works. The last term in β corresponds to a correction $\simeq \lambda(\lambda\omega^2)$ in this energy difference and may not be ignored like in the case of the self-energy. In the equal-time case (3) would be required to be *added* as an external constraint to the theory based upon physical considerations. It will ensure that the sum of the tadpole diagrams, to the approximation concerned, for the transition $\varphi \rightarrow$ *vacuum* vanishes. *The physical outcome would then be the same in the two frameworks of treating the problem considered.* The variational methods write only the first two ($\approx$ tree level) terms in the expression for β and thus ignore the terms coming from the finite corrections. A similar remark can be made about the last term in (2).

Consider first the *symmetric phase* with $\omega \approx 0$, which is allowed by (3). From (2) we compute $\partial M^2/\partial\omega = 2\lambda\omega(3 - \sqrt{3}\lambda/M^2)/[1 + 3\lambda/(4\pi M^2) - \sqrt{3}\lambda^2\omega^2/M^4]$ which is needed to find $\beta' \equiv d\beta/d\omega = d^2F/d\omega^2$. Its sign will determine the nature of the stability of the vacuum. We find $\beta'(\omega = 0) = M^2[1 - 0.0886(\lambda/M^2)^2]$, where by the same arguments as made above in the case of β we may not ignore the λ^2 term. The β' changes the sign from a positive value for vanishingly weak couplings to a negative one when the coupling increases. In other words the system starts out in a *stable symmetric phase* for vanishingly small coupling but passes over into an *unstable symmetric phase* for values greater than $g_s \equiv \lambda_s/(2\pi m^2) \simeq 0.5346$.

Consider next the case of *the spontaneously broken symmetry phase* ($\omega \neq 0$). From (3) the values of ω are now given by $M^2 - 2\lambda\omega^2 + \sqrt{3}\lambda/2 = 0,$ where we have used the tree level approximation $2\lambda\omega^2 \simeq M^2$. The mass renormalization condition becomes $M^2 - m^2 = 3\lambda\omega^2 + (3\lambda/(4\pi))\, ln(m^2/M^2) - \lambda\sqrt{3}/2$. On eliminating ω we obtain the *modified duality relation*

$$\frac{1}{2}M^2 + m^2 + \frac{3\lambda}{4\pi}\, ln(\frac{m^2}{M^2}) + \frac{\sqrt{3}}{4}\lambda = 0. \qquad (5)$$

which can also be rewritten as $[\lambda\omega^2 + m^2 + (3\lambda/(4\pi))ln(m^2/M^2)] = 0$ and it shows that the real solutions exist only for $M^2 > m^2$. The finite corrections found here are

again not considered in the earlier works, for example, they assume (or find) the tree level expression $M^2 - 2\lambda\omega^2 = 0$. In terms of the dimensionless coupling constants $g = \lambda/(2\pi m^2) \geq 0$ and $G = \lambda/(2\pi M^2) \geq 0$ we have $G < g$. The new self-duality eq. differs from the old one and shifts the critical coupling to a higher value. We find that: *i)* for $g < g_c = 6.1897$ (*critical coupling*) there is no real solution for G, *ii)* for a fixed $g > g_c$ we have two solutions for G one with the point lying on the upper branch ($G > 1/3$) and the other with that on the lower branch ($G < 1/3$), of the curve describing G as a function of g and which starts at the point ($g = g_c = 6.1897, G = 1/3$), *iii)* the lower branch with $G < 1/3$, approaches to a vanishing value for G as $g \to \infty$, in contrast to the upper one for which $1/3 < G < g$ and G continues to increase. From $\beta(\omega) \approx \omega[M^2 - 2\lambda\omega^2 + \sqrt{3}\lambda/2]$ and (5) we determine $\beta' \approx (1 + 0.9405G)$ which is always positive and thus indicates a minimum for the difference $F(\omega)$ of the vacuum energy densities (when ω is nonvanishing). The energetically favored broken symmetry phases become available only after the coupling grows to the critical coupling $g_c = 6.18969$ and beyond this the asymmetric phases would be preferred against the unstable symmetric phase in which the system finds itself when $g > g_s \simeq 0.5346$. The phase transition is thus of the *second order* confirming the conjecture of Simon-Griffiths. If we ignore the additional finite renormalization corrections we obtain complete agreement with the earlier results and a first order phase transition, e.g., the symmetric phase always remains stable but for $g > 1.4397$ the energetically favoured symmetric phases also do appear. From numerical computation we verify that at the minima corresponding to the nonvanishing value of ω the value of F is negative and that for a fixed g it is more negative for the point on the lower branch ($G < 1/3$) than for that on the upper branch ($G > 1/3$).

3. The present work and the earlier one on the mechanism of *SSB* add to the previous experience that the *front form* dynamics is a useful complementary method and needs to be studied systematically in the context of QCD and other problems. The physical results following from one or the other form of the theory should come out to be the same though the mechanisms to arrive at them may be different. In the equal-time case we are required to add external considerations in order to constrain the theory while the

analogous conditions in the light-front formulation seem to be already contained in it through the self-consistency equations. When additional fields, e.g., fermionic ones are present also the constraint equations in the theory quantized on the light-front would relate the various types of vacuum condensates (vacuum expectation values of composite scalar fields).

References:

a. Shortened from Nuovo Cimento A, 1994; also one submitted to Physics Letters B.

[1.] The Hamiltonian (3-1) and the constraint eq. (4-1) used here derived in the previous contribuition (hep-th@xxx.lanl.gov/9412204) *Light-front quantization and Spontaneous symmetry breaking* are

$$P^- = \int dx \left[\omega(\lambda\omega^2 - m^2)\varphi + \frac{1}{2}(3\lambda\omega^2 - m^2)\varphi^2 + \lambda\omega\varphi^3 + \frac{\lambda}{4}\varphi^4\right] \tag{3}$$

$$\begin{aligned} & lim_{L\to\infty}\frac{1}{L}\int_{L/2}^{L/2} dx\, V'(\phi) \equiv \\ & \omega(\lambda\omega^2 - m^2) + lim_{L\to\infty}\frac{1}{L}\int_{-L/2}^{L/2} dx\Big[(3\lambda\omega^2 - m^2)\varphi + \lambda(3\omega\varphi^2 + \varphi^3)\Big] = 0 \end{aligned} \tag{4}$$

[2.] B. Simon and R.B. Griffiths, Commun. Math. Phys. **33** (1973) 145; B. Simon, *The $P(\Phi)_2$ Euclidean (Quantum) Field Theory*, Princeton University Press, 1974.

[3.] See for list of references: *Lectures on Light-front quantized field theory*, Proceedings *XIV Brazilian National Meeting on Particles and Fields*, Sociedade Barasileira de Física, pgs. 154-192, 1993. Available also from hep-th@xxx.lanl.gov, no. 9312064.